Critical temperature for α-particle condensation in asymmetric nuclear matter.

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The critical temperature for α-particle condensation in nuclear matter with Fermi surface imbalance between protons and neutrons is determined. The in-medium four-body Schrödinger equation, generalizing the Thouless criterion of the BCS transition, is applied using a Hartree-Fock wave function for the quartet projected onto zero total momentum in matter with different chemical potentials for protons and neutrons.

PACS numbers: 21.65.Cd, 67.85.Lm

I. INTRODUCTION

Clustering in nuclei and nuclear matter has become an increasingly studied subject recently [1–14]. Since the α-particle is the smallest doubly magic nucleus, it is thus very stable and excited heavier nuclei may exist in weakly bound states consisting of nucleons, pairs, α-particles, etc. [6, 7, 12]. Recently, excited states of ^{12}C and ^{16}O, like the Hoyle state in ^{12}C, have been described as so-called "α condensation" states, which can be pictured by three or four α-particles occupying the lowest s-state in an effective quartet mean-field potential [1, 11].

On the other hand, α-clustering in nuclear matter has also been investigated as a possible phase in compact stars or as a precursor of α-clustering in finite nuclei. The critical temperature for the α condensation has previously been calculated with the momentum projected Hartree-Fock approximation, yielding results consistent with the one by the Faddeev-Yakubovsky method [14]. The critical temperature is an important quantity with respect to stellar nucleosynthesis and star formation [15, 16]. However, in comparison with our previous studies, bulk nuclear matter, like the one in neutron stars and other compact objects, is imbalanced with respect to the particle numbers of the protons and neutrons, implying different chemical potentials. In the present paper, we investigate the critical temperature in asymmetric nuclear matter.

The applied method is the same as in our previous work of Ref. [14], where the in-medium four-body Schrödinger equation is computed employing Hartree-Fock wave functions under the constraint of total momentum equal to zero. While we used four equal single particle wave functions in the previous paper for symmetric nuclear matter, we shall employ different wave functions for protons and neutrons in the present case because of the population imbalance with the different chemical potentials for protons and neutrons.

In the next section, we shortly will describe the formulation which we use to compute the critical temperature. In Section III, we show the numerical results for the critical temperature as a function of density and chemical potential for various population ratios of protons versus neutrons. Besides, we will display and discuss the quartet wave function at the critical temperature. Finally in Section IV, we summarize.

II. FORMULATION

For the asymmetric nuclear matter case, we will closely follow the formulation given in our previous paper [14] for symmetric nuclear matter.

The Hamiltonian is represented by

$$H = H_0 + V = \sum_1 \varepsilon_1 c_1^\dagger c_1 + \frac{1}{4}\sum_{1234} \bar{v}_{12,34} c_1^\dagger c_1^\dagger c_4 c_3, \quad (1)$$

where indices represent momentum, spin, and isospin, $\bar{v}_{12,1'2'}$ is the antisymmetric two-body interaction matrix, and $\varepsilon_i = \frac{k_i^2}{2m^*}$. We formally introduce an effective mass m^* to account approximately for the exchange term of the mean field, whereas the direct term is incorporated into the chemical potential. However, since in the following we consider mostly only systems at very low density we will disregard mean field effects and take for the mass the bare one.

The four-body Schrödinger equation in the medium with the eigenvalue E is given by (see [14, 17–19] for details)

$$\varepsilon_{1234}\psi_{1234} + \sum_{1'2'3'4'} V_{1234;1'2'3'4'}\psi_{1'2'3'4'} = E\psi_{1234} \quad (2)$$

where $\varepsilon_{1234} = \sum_{i=1}^4 \varepsilon_i$ and $V_{1234;1'2'3'4'}$ is of the form

$$V_{1234;1'2'3'4'} = (1 - f_1 - f_2)\frac{1}{2}\bar{v}_{12,1'2'}\delta_{33'}\delta_{44'}$$

$$+(1-f_1-f_3)\frac{1}{2}\bar{v}_{13,1'3'}\delta_{22'}\delta_{44'} + \text{permutation}, \quad (3)$$

where f_i is the Fermi-Dirac distribution with different chemical potentials for protons and neutrons. Otherwise, formally the in-medium four-body equation is exactly the same as in the symmetric case. The condition of the transition to condensation, known as the Thouless criterion, is satisfied with $E = 2\mu_p + 2\mu_n$ at $T \to T_c$ [20], where μ_p (μ_n) is the proton (neutron) chemical potential.

For simplicity, we consider also for proton-neutron imbalanced nuclear matter a spin-isospin-independent two body interaction. We do not think that this is much less justified than in the symmetric case [14] where this was very successful, indeed. We again employ for the four body wave function ψ_{1234} the Hartree-Fock ansatz, projected on zero total momentum:

$$\begin{aligned}\psi_{1234} &\to \varphi_p(\vec{k}_1)\varphi_p(\vec{k}_2)\varphi_n(\vec{k}_3)\varphi_n(\vec{k}_4)\chi_0 \\ &\times (2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4) \end{aligned} \quad (4)$$

where $\varphi_\tau(\vec{k}_i) = \varphi_\tau(|\vec{k}_i|)$ is the s-wave single particle wave functions for protons ($\tau = p$) and neutrons ($\tau = n$), respectively. χ_0 is the spin-isospin singlet wave function.

Substituting the ansatz of Eq. (4) into Eq. (2), and integrating over superfluous variables, we obtain

$$\varphi_\tau(k) = \frac{-3\mathcal{B}_\tau(k)}{\mathcal{A}_\tau(k) + 3\mathcal{C}_\tau(k)}, \quad (\tau = p, n) \quad (5)$$

where

$$\begin{aligned}\mathcal{A}_p(k) &= \int \frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\frac{d^3k_4}{(2\pi)^3} \\ &\times\left(\frac{k^2}{2m}+\frac{k_2^2}{2m}+\frac{k_3^2}{2m}+\frac{k_4^2}{2m}-2\mu_p-2\mu_n\right) \\ &\times(\varphi_p(\vec{k}_2))^2(\varphi_n(\vec{k}_3))^2(\varphi_n(\vec{k}_4))^2 \\ &\times(2\pi)^3\delta(\vec{k}+\vec{k}_2+\vec{k}_3+\vec{k}_4), \end{aligned} \quad (6)$$

$$\begin{aligned}\mathcal{B}_p(k) &= \int \frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\frac{d^3k_4}{(2\pi)^3}\frac{d^3k_1'}{(2\pi)^3}\frac{d^3k_2'}{(2\pi)^3} \\ &\times(1-f_p(k)-f_p(k_2))v_{\vec{k}\vec{k}_2,\vec{k}_1'\vec{k}_2'} \\ &\times(2\pi)^3\delta(\vec{k}+\vec{k}_2-\vec{k}_1'-\vec{k}_2') \\ &\times\varphi_p(\vec{k}_1')\varphi_p(\vec{k}_2)\varphi_p(\vec{k}_2')(\varphi_n(\vec{k}_3))^2(\varphi_n(\vec{k}_4))^2 \\ &\times(2\pi)^3\delta(\vec{k}+\vec{k}_2+\vec{k}_3+\vec{k}_4) \\ &+ 2\int \frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\frac{d^3k_4}{(2\pi)^3}\frac{d^3k_1'}{(2\pi)^3}\frac{d^3k_3'}{(2\pi)^3} \\ &\times(1-f_p(k)-f_n(k_3))v_{\vec{k}\vec{k}_3,\vec{k}_1'\vec{k}_3'} \\ &\times(2\pi)^3\delta(\vec{k}+\vec{k}_3-\vec{k}_1'-\vec{k}_3') \\ &\times\varphi_p(\vec{k}_1')(\varphi_p(\vec{k}_2))^2\varphi_n(\vec{k}_3)\varphi_n(\vec{k}_3')(\varphi_n(\vec{k}_4))^2 \\ &\times(2\pi)^3\delta(\vec{k}+\vec{k}_2+\vec{k}_3+\vec{k}_4), \end{aligned} \quad (7)$$

$$\begin{aligned}\mathcal{C}_p(k) &= \int \frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\frac{d^3k_4}{(2\pi)^3}\frac{d^3k_3'}{(2\pi)^3}\frac{d^3k_4'}{(2\pi)^3} \\ &\times(1-f_n(k_3)-f_n(k_4))v_{\vec{k}_3\vec{k}_4,\vec{k}_3'\vec{k}_4'} \\ &\times(2\pi)^3\delta(\vec{k}_3+\vec{k}_4-\vec{k}_3'-\vec{k}_4') \\ &\times(\varphi_p(\vec{k}_2))^2\varphi_n(\vec{k}_3)\varphi_n(\vec{k}_3')\varphi_n(\vec{k}_4)\varphi_n(\vec{k}_4') \\ &\times(2\pi)^3\delta(\vec{k}+\vec{k}_2+\vec{k}_3+\vec{k}_4) \\ &+ 2\int \frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\frac{d^3k_4}{(2\pi)^3}\frac{d^3k_2'}{(2\pi)^3}\frac{d^3k_3'}{(2\pi)^3} \\ &\times(1-f_p(k_2)-f_n(k_3))v_{\vec{k}_2\vec{k}_3,\vec{k}_2'\vec{k}_3'} \\ &\times(2\pi)^3\delta(\vec{k}_2+\vec{k}_3-\vec{k}_2'-\vec{k}_3') \\ &\times\varphi_p(\vec{k}_2)\varphi_p(\vec{k}_2')\varphi_n(\vec{k}_3)\varphi_n(\vec{k}_3')(\varphi_n(\vec{k}_4))^2 \\ &\times(2\pi)^3\delta(\vec{k}+\vec{k}_2+\vec{k}_3+\vec{k}_4) \end{aligned} \quad (8)$$

with the symmetric two-body vertex $v_{\vec{k}_2\vec{k}_3,\vec{k}_2'\vec{k}_3'}$. The corresponding expressions for the neutrons $\mathcal{A}_n$, $\mathcal{B}_n$, and $\mathcal{C}_n$ are obtained by exchanging indices $p \leftrightarrow n$ for $\mathcal{A}_p$, $\mathcal{B}_p$, and $\mathcal{C}_p$ in Eqs. (6), (7) and (8). The Fermi distribution functions are

$$f_\tau(k) = \frac{1}{e^{(\frac{k^2}{2m}-\mu_\tau)/T}+1}, \quad (\tau = p, n). \quad (9)$$

Here, comparing with symmetric nuclear matter [14], in the imbalanced nuclear matter case, two coupled equations are obtained for protons and neutrons.

III. NUMERICAL CALCULATION

As seen from Eqs. (6)-(8), since the wave functions $\varphi_{p,n}(k)$ are mixed up in $\mathcal{A}_{p,n}$, $\mathcal{B}_{p,n}$, and $\mathcal{C}_{p,n}$, Eq. (5) constitutes in fact two coupled non linear equations to be solved self-consistently by iteration. The critical temperature is derived from the condition

$$\begin{aligned}&\int \frac{d^3k}{(2\pi)^3}\varphi_p(k)[(\mathcal{A}_p(k)+3\mathcal{C}_p(k))\varphi_p(k)+3\mathcal{B}_p(k)] \\ &= 0, \end{aligned} \quad (10)$$

$$\begin{aligned}&\int \frac{d^3k}{(2\pi)^3}\varphi_n(k)[(\mathcal{A}_n(k)+3\mathcal{C}_n(k))\varphi_n(k)+3\mathcal{B}_n(k)] \\ &= 0. \end{aligned} \quad (11)$$

Given a set of chemical potentials μ_p and μ_n one can solve the two coupled equations (10) and (11) in adjusting the temperature, to be identified with the critical temperature, T_c, as the single parameter.

For the vertex $v_{\vec{k}_1\vec{k}_2,\vec{k}_1'\vec{k}_2'}$ in Eqs. (7) and (8), we again take the separable potential of [14]:

$$\begin{aligned}v_{\vec{k}_1\vec{k}_2,\vec{k}_1'\vec{k}_2'} &= \lambda e^{-\frac{(\vec{k}_1-\vec{k}_2)^2}{4b^2}}e^{-\frac{(\vec{k}_1'-\vec{k}_2')^2}{4b^2}} \\ &\times (2\pi)^3\delta(\vec{k}_1+\vec{k}_2-\vec{k}_1'-\vec{k}_2'). \end{aligned} \quad (12)$$

The parameters λ and b are adjusted to the binding energy (-28.3MeV) and to the rms radius (1.71fm) of the isolated α-particle; $\lambda = -992$MeV fm^3 and $b = 1.43$fm^{-1}. As already mentioned, in the symmetric case our procedure to solve the in-medium four-body equation with our

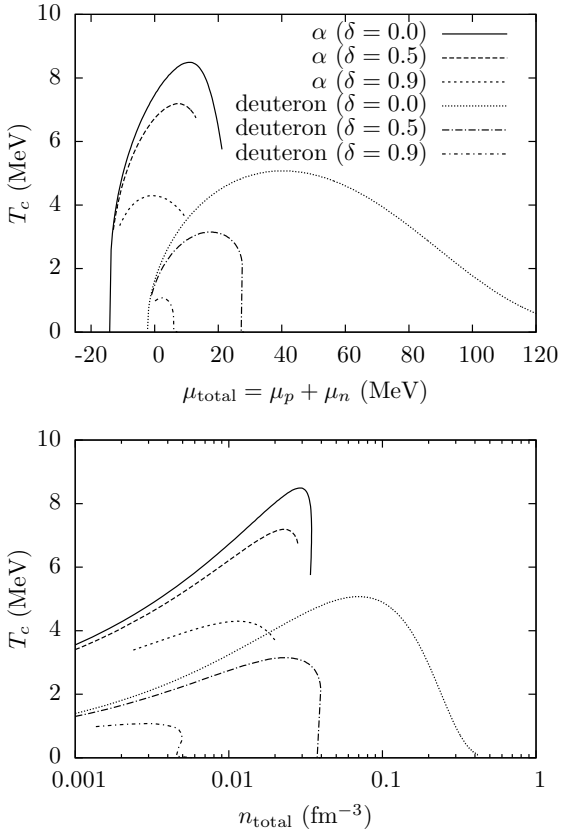

FIG. 1: Critical temperature as functions of the total chemical potential $\mu_{\rm total} = \mu_p + \mu_n$ (top) and the total free density $n_{\rm total}$ (bottom). The density ratio δ is in Eq. (13).

ansatz (4) and the separable interaction (12) gave excellent agreement with a full Faddeev-Yakubovsky solution using the Malfliet-Tjon interaction (MT I-III) [14]. We, therefore, think that our procedure is valid in the present case as well.

Fig. 1 shows the critical temperature of α condensation as a function of the total chemical potential $\mu_{\rm total} = \mu_p + \mu_n$. We see that T_c decreases as the asymmetry, given by the parameter

$$\delta = \frac{n_n - n_p}{n_n + n_p}, \tag{13}$$

increases. This is in analogy with the deuteron case (also shown) which already had been treated in [21, 22]. On the other hand it also is interesting to show T_c as a function of the free density which is

$$n_{\rm total} = n_p + n_n \tag{14}$$

$$n_p = 2\int \frac{d^3k}{(2\pi)^3} f_p(k) \tag{15}$$

$$n_n = 2\int \frac{d^3k}{(2\pi)^3} f_n(k), \tag{16}$$

where the factor two in front of the integral comes from the spin degeneracy, and $f_{p,n}(k)$ is as in Eq. (9). It should be emphasized, however, that in the above relation between density and chemical potential, the free gas relation is used and correlations in the density have been neglected. In this sense the dependence of T_c on density only is indicative, more valid at the higher density side. The very low density part where the correlations play a more important role shall be treated in a future publication. It should, however, be stressed that the dependence of T_c on the chemical potential as in the upper panel if Fig. 1, stays unaltered.

The fact that for more asymmetric matter the transition temperature decreases, is natural, since as the Fermi levels become more and more unequal, the proton-neutron correlations will be suppressed. For small δ's, i.e., close to the symmetric case, α condensation (quartetting) breaks down at smaller density (smaller chemical potential) than deuteron condensation (pairing). This effect has already been discussed in our previous work for symmetric nuclear matter [14, 17]. For large δ's, i.e. strong asymmetries, the behavior is opposite, i.e., deuteron condensation breaks down at smaller densities than α condensation, because the small binding energy of the deuteron can not compensate the difference of the chemical potentials.

More precisely, for small δ's, the deuteron with zero center of mass momentum is only weakly influenced by the density or the total chemical potential as can seen in Fig. 1. However, as δ increases, the different chemical potentials for protons and neutrons very much hinders the formation of proton-neutron Cooper pairs in the isoscalar channel for rather obvious reasons. The point to make here is that because of the much stronger binding per particle of the α-particle, the latter is much less influenced by the increasing difference of the chemical potentials. For the strong asymmetry $\delta = 0.9$ in Fig. 1 then finally α-particle condensation can exist up to $n_{\rm total} = 0.02{\rm fm}^{-3}$ ($\mu_{\rm total} = 9.3$MeV), while the deuteron condensation exists only up to $n_{\rm total} = 0.005{\rm fm}^{-3}$ ($\mu_{\rm total} = 6.0$MeV).

Overall, the behavior of T_c is more or less as can be expected. We should, however, remark that the critical temperature for α-particle condensation stays quite high, even for the strongest asymmetry considered here, namely $\delta = 0.9$. This may be of importance for the possibility of α-particle condensation in neutron stars and supernovae explosions [23, 24].

We also show the single particle wave functions of protons and neutrons, entering the quartet wave function (4), for various ratios of Fermi surface imbalance and chemical potentials in Fig. 2. In most cases of Fig. 2, the momentum-space wave functions with negative chemical potentials are monotonically decreasing whereas the ones with positive chemical potentials have a dip at $k = 0$. However, the momentum-space wave functions also develop a dip at $k = 0$ even at a negative chemical potential as the asymmetry takes on stronger values. This can be seen, e.g. for $\delta = 0.5$, $\mu_{\rm total} = 8.93$MeV, and $\delta = 0.9$, $\mu_{\rm total} = -11.0$MeV in Fig. 2. Furthermore, the neutron wave function in k-space with large positive chemical po-

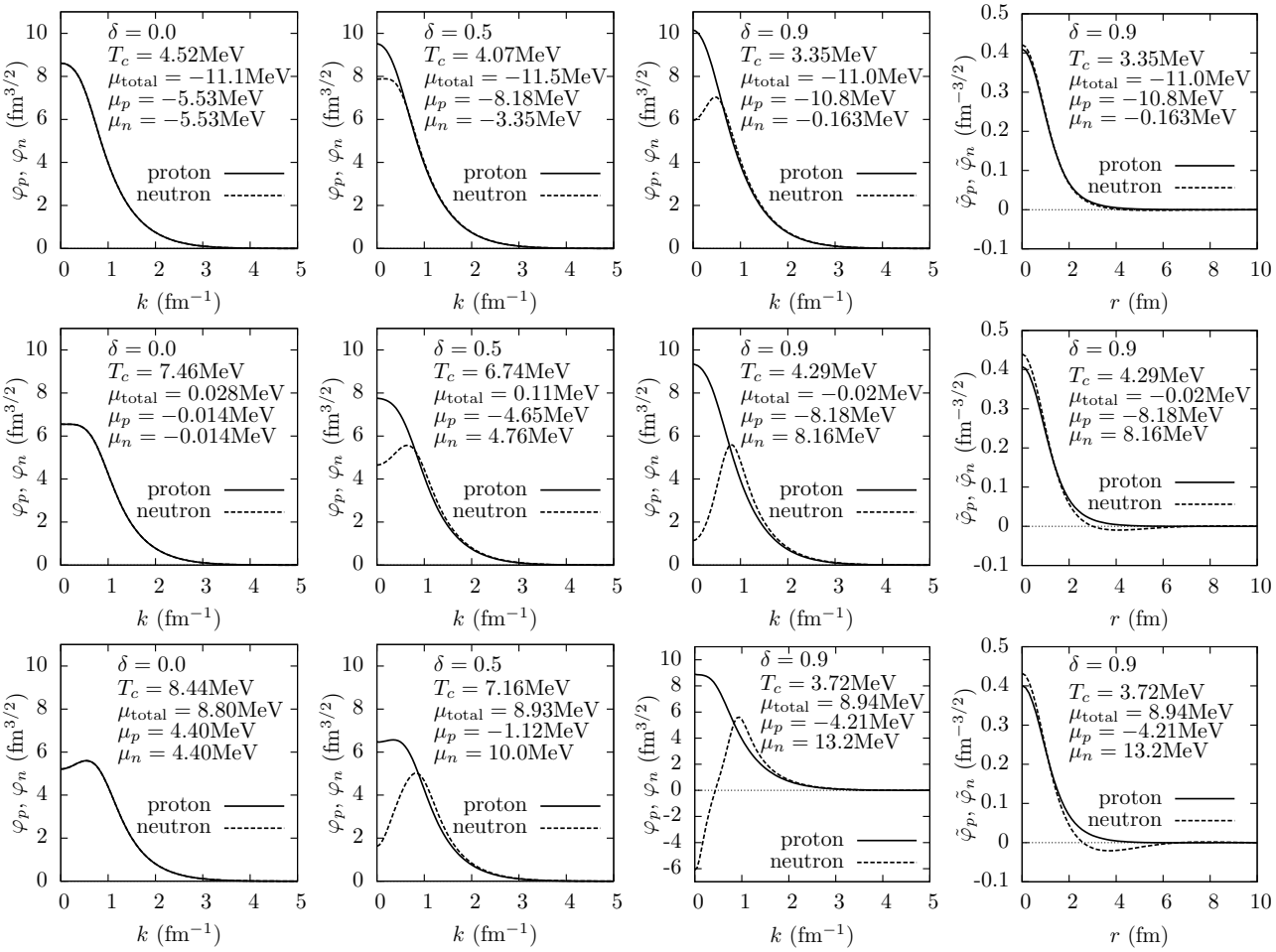

FIG. 2: The momentum-space single particle wave functions φ_p, φ_n at the critical temperature T_c as function of k for $\delta = 0.0$, 0.5 0.9, and the real-space wave functions $\tilde{\varphi}_p$, $\tilde{\varphi}_n$ as a function of r for $\delta = 0.9$ derived from the Fourier transform of $\varphi_{p,n}(k)$ with $\tilde{\varphi}_{p,n}(r) = \int d^3k e^{i\vec{k}\cdot\vec{r}}\varphi_{p,n}(k)/(2\pi)^3$. The top, middle and bottom figures are for $\mu_{\rm total} = \mu_p + \mu_n \sim -11$MeV, ~ 0.0MeV, and ~ 9.0MeV, respectively. The wave functions are normalized by $\int d^3k\varphi^2_{p,n}(k)/(2\pi)^3 = 1$.

tential develops a node. This behavior is similar to the wave functions in Ref. [14]. As shown in Fig. 2, the dissymmetry of proton and neutron wave functions increases as δ increases. As a consequence, the critical temperature decreases, and the α condensation breaks down at a more dilute density, see Fig. 1. We also present in most right figures of Fig. 2 the proton and neutron wave functions in real space. In spite of the sometimes strong dissymmetry in momentum space, the proton and neutron wave functions are relatively more similar to one another in r-space. The neutron wave function develops a node as the total chemical potential $\mu_{\rm total} = \mu_p + \mu_n$ increases, but the negative values of the wave function remain rather moderate.

IV. SUMMARY

We reported on the critical temperature for the α-particle condensation as a function of the density and chemical potential in asymmetric nuclear matter. The four-body wave function in the medium is calculated with Hartree-Fock wave functions projected onto zero total momentum, a procedure which was already very successful in the symmetric case. Not unexpectedly the transition temperature decreases with increasing asymmetry. However, it was shown that T_c stays relatively high for very strong asymmetries, a fact of importance in the astrophysical context. The single particle wave functions of proton and neutron were also shown and discussed. The neutron wave function in momentum space develops a node for strong asymmetries and high densities, a fact familiar from ordinary pairing. It also was shown that asymmetry affects deuteron pairing more strongly than α-particle condensation. Therefore, at high asymmetries,

if at all, α-particle condensation dominates over pairing at all possible densities.

The condition of Eqs. (10) and (11) corresponds to calculating the critical temperature at the point where the α-particles in nuclear matter dissociate into four free particles, i.e. at the Mott transition temperature. In the higher densities considered here, this temperature is consistent with the phase transition temperature between superfluid and normal phase. In the strong coupling limit, this is not the case, since the Bose Einstein condensation breaks down at lower temperature than the one of dissociation of a bound state to free particles. This problem in the BCS-BEC crossover can be solved by the Nozières and Schmitt-Rink theory, where the pair fluctuations are systematically included beyond the mean field approximation [25]. The extension of this theory to the quartet condensation is difficult but will be attempted in future work.

Acknowledgments

This work is supported by DFG No. RO905/29-2.

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